101 Domino-tions? (P398). For the purposes of this problem, a domino is simply a 1×2 rectangle that must be placed in the plane so that all of its vertices have integer coordinates. What is the smallest rectangle, by perimeter, into which 101 nonoverlapping dominoes may be placed so that no two dominoes share an entire long edge?

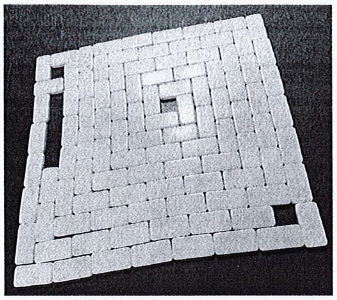

Figure 3. 101 dominos arranged in a rectangle.

We received solutions from Gabriel Augusto Correia (Universidade de Brasília), Evan Ganning and Meagan Praul (Seton Hall University), Tucker Germain, Kabrie Karschner, and Micah Wheeler (Taylor University), Vasile Teodorovici (NSERC Canada), Stan Wagon (Macalaster College), and the Armstrong, Cal Poly Pomona, Missouri State University, and Skidmore College problem-solving groups. We also received a partial solution from Josh Harden, Lauren James, and Autumn Thompson of Taylor University.

The minimum possible perimeter is 58. To see this, note first that the rectangle must have sufficient area to fit all 101 dominos, or at least 202 square units. The minimum perimeter rectangle for a given area is a square, but a 14×14 square (with perimeter 56) is slightly too small. Thus, the smallest possible perimeter is 58, and indeed, as the photo in figure 3 from Meagan and Evan shows, it is possible to pack all 101 dominos into a 14×15 rectangle with this perimeter. The groups from Armstrong State, Missouri State, and Seton Hall also noted it is possible to pack them into a 13×16 rectangle, and that a 12×17 rectangle apparently has barely enough area, but is nevertheless unworkable because of the long-side non-adjacency rule.

Pyramix (P400). In his article "Tricolor Pyramids," Jacob Siehler investigated three-colored triangular arrays in which every triad composed of a cell and the two below it is either monochrome or contains all three colors. Suppose a size n pyramid is colored in a non-monochrome fashion according to this rule. Find, with proof, the maximum number of any one color of cells that the pyramid can contain.

We received the solution presented from the Missouri State Problem Solving Group, and partial solutions from Randy Schwartz and from the Skidmore College problem solvers. On the one hand, a non-monochrome triangle can have at most $n(n-1)/2$ cells of one color. Note that by making the top $n-1$ rows monochrome and alternating the other two colors on the bottom row, we can achieve a non-monochrome triangle of size n with $n(n-1)/2$ cells of one color.

On the other hand, if there are $1+n(n-1)/2$ cells of color c, then there is at least one row entirely colored c (as otherwise at most $0+1+2+\cdots+(n-2)+(n-1)=n(n-1)/2$ cells can be color c). So, let k be the longest row that is monochrome colored c. All of the shorter rows must be color c, by the coloring rule, and none of row $(k+1)$ can be color c (or else they would all be c). If $k < n$, that would leave

$$1+(k+1)+(k+2)+\cdots+(n-2)+(n-1)$$

cells of color c in $(k+2)+(k+3)+\cdots+(n-1)+n$ positions, so again we would have to have a (longer) row entirely colored c, contradicting the choice of k. We conclude $k = n$, i.e., the triangle is monochrome.

Equitable Marbles (P401). This problem was submitted by Matt Enlow (Dana Hall School, Wellesley, MA).

I have a bag of 100 marbles of three different colors. If you were to reach in and grab three of the marbles at random, there's a 20% chance that you would pull out one of each color. How many marbles of each color are in my bag?

The Playground received a baker's dozen responses to this problem, including solutions from Michael Ask (Ulysses High School), Brian Beasley, Dmitry Fleischman, Tarlan Ismayilsoy (ADA University, Azerbaijan), Kayla Nicolich and Leslie Rodriguez (Seton Hall University), Randy Schwartz, Vasile Teodorovici, and the Armstrong, Cal Poly Pomona, Missouri State, and Skidmore College problem-solving groups, as well as partial solutions from two groups of collaborators at Taylor University: Sarah Gorski, Rebekah Griggs, and Flora Wang; and Jacob Hockett, Scott Mitchell, and Clay Vander Kolk.

There are **21, 35,** and **44** marbles of the three colors. Largely following the (similar) Armstrong and Missouri solutions, let a, b, and c represent the numbers of marbles of each color, so that $a + b + c = 100$. There are abc ways to choose one marble of each color, which must account for 1/5 of the

$$\binom{100}{3} = 161{,}700$$

ways to choose any three marbles, meaning $abc = 32{,}340$.

For fixed a, the product bc is maximized when $b = c$, whence $a(100-a)^2 \geq 4 \cdot 32{,}340$. By color symmetry, this bounds all of a, b, and c between 21 and 48, inclusive. Now note the prime factorization of $32{,}340 = 2^2 \cdot 3 \cdot 5 \cdot 7^2 \cdot 11$; each of these prime factors must be distributed into one of a, b, or c.

Because a, b, and c are less than 49, we immediately have that 7, 7, and 11 must be distributed into different summands. Then by considering where the factor of 5 goes, one of the summands must be exactly 35, leaving the other two with product 924 and sum 65. This latter pair has the unique solution {21, 44}, as desired.

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